

Exercise 71

Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

Solution

Differentiate $r(x)$.

$$\begin{aligned}r'(x) &= \frac{d}{dx}[r(x)] \\&= \frac{d}{dx}[f(g(h(x)))] \\&= f'(g(h(x))) \cdot \frac{d}{dx}[g(h(x))] \\&= f'(g(h(x))) \cdot g'(h(x)) \cdot \frac{d}{dx}[h(x)] \\&= f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)\end{aligned}$$

Evaluate it at $x = 1$.

$$\begin{aligned}r'(1) &= f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) \\&= f'(g(2)) \cdot g'(2) \cdot (4) \\&= f'(3) \cdot (5) \cdot (4) \\&= (6) \cdot (5) \cdot (4) \\&= 120\end{aligned}$$